

Preference Similarity Network Structural Equivalence Clustering based Consensus Group Decision Making Model

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Abstract

Social network analysis (SNA) methods have been developed to analyse social structures and patterns of network relationships, although they have been least explored and/or exploited purposely for decision-making processes. In this study, we bridge a gap between SNA and consensus-based decision making by defining undirected weighted preference network from the similarity of expert preferences using the concept of ‘structural equivalence’. Structurally equivalent experts are represented using the agglomerative hierarchical clustering algorithm with complete link function, thus intra-clusters’ experts are high in density and inter-clusters’ experts are rich in sparsity. We derive cluster consensus based on internal and external cohesions, while group consensus is obtained by identifying the highest level consensus at optimal level of clustering. Thus, the clustering based approach to consensus measure contributes to present homogeneity of experts preferences as a whole. In the event of insufficient group consensus state, we construct a feedback mechanism procedure based on clustering that consists of three main phases: (1) identification of experts that contribute less to consensus; (2) identification of a leader in the network; and (3) advice generation. We make use of the centrality concept in SNA as a way of determining the most important person in a network, who is presented as a leader to provide advices in the feedback process. It is proved that the implementation of the proposed feedback mechanism increases consensus and, because of the bounded condition of consensus measure, convergence to sufficient group agreement is guaranteed. The centrality concept is also applied in the construction of a new aggregation operator, namely as cent-IOWA operator, that is used to derive the collective preference relation from which the feasible alternative of consensus solution, based on the concept of dominance, is achieved according to a majority of the central experts in the network, which is represented in this paper by the linguistic quantifier ‘*most of*.’ For validation purposes, an existing literature study is used to perform a comparative analysis from which conclusions are drawn and explained.

Keywords: Consensus Group Decision Making, Social Network Analysis, Opinion Similarity, Structural Equivalence, Agglomerative Hierarchical Clustering, IOWA-based aggregation operator.

1. Introduction

In decision making, experts can use different representation formats to provide their opinions or preferences on a set of alternatives, whether in numerical or linguistic form. One of the well-known representation formats is the preference relation, which is based on alternatives pairwise comparisons. The concept of reciprocal preference relation to represent intensities of preferences was proposed by Bezdek et al. [1], comprehensively interpreted in Nurmi [2] and broadly studied

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in Chiclana et al. [3], Wu and Chiclana [4–6], Wu and Xu [7], Perez-Asurmendi and Chiclana [8], Urena et al. [9] and Liu et al. [10]. Most of these studies focused on the properties and the implementation of reciprocal preference relations in decision making and consensual reaching process perspectives.

When dealing with preferences of experts in consensus processes, it is necessary to measure how close experts' preferences are [11]. Similarity functions are thus implemented in consensus processes to represent the concept of closeness between preferences on each pair of experts and each pair of available alternatives [12]. A comparative analysis of different similarity functions in a consensus reaching process was carried out by Chiclana et al. in [13].

Another important issue in group decision making (GDM) that has to be taken into account before making the final decision is the level of group agreement. Indeed, some experts may not accept the final decision made if they consider their individual preferences have not been considered appropriately [14, 15]. Thus, it is worth to suggest that experts should engage in a consensus process where they can discuss and change their preferences to make them closer to each other using an appropriate feedback mechanism procedure with the purpose of obtaining a high level of group agreement. One way of executing feedback mechanisms is via a consensus moderator who would act as an advisor to all individual experts on how to modify their preferences while controlling, at the same time, the consensus state of the group of experts as a collective [13]. New consensus approaches based on interactive tools or systems have been developed in an attempt to substitute the role of moderator with the aim to make the consensus process to be carried out automatically [16–22].

Most GDM and consensus models deal with a few number of experts, because normally important decisions are only made by professional, skilful and authorized persons in the companies, administrations or organizations. However, current electronic technology and society demands lead us to large-scale group decision making paradigms, like *social networks* (*Facebook*, *Instagram*, *Twitter*, etc.) and *Web 2.0* (*Wikipedia*, *Amazon* online store, blogs, forums, etc.). Previous researchers have analysed the relationship within these social networking web-sites, with a predominant focus on the structures and patterns of the social network analysis (SNA). However, network interactions between the individual and group of experts involved in a decision-making process is one of the least investigated SNA methodology driven research areas. In recent years, researchers have come up with the idea of combining SNA properties such as centrality, adjacency, trust statements, in the development of decision making or consensual models, as presented in Wu and Chiclana [4], Perez et al. [23, 24], Wu et al. [25], Brunelli et al. [26] and Chu et al. [27], Dong et. al [28].

It is common to exchange opinions through interaction in a network; however it will be difficult to get high level of consensus when it involves a large number of users. Generally, this situation can be resolved by categorizing users into subgroups, thus they can be effectively treated in small numbers. One of the most widely used methodology to partition objects is clustering [29–33]. In the consensus based decision making context, this approach allows to partition a set of experts into subgroups or *clusters* based on similarity of preferences, where some of them are placed in the same cluster because they are more similar to each other than with experts in different cluster(s). Some research efforts on this area have been presented in Perony et al. [34], Garcia-Lapresta and Perez-Roman [35–37], Abel et al. [38] and Li et al. [39].

In the area of the information fusion, the introduction of the Ordered Weighted Averaging (OWA) [40] and the Induced OWA (IOWA) [41] operators allowed many researchers to propose extended versions of OWA/IOWA-based operators to perform aggregation of information [42–48]. Because the fundamental aspect of IOWA-based operators is the reordering of the argument to be aggregated by means of additional order inducing variables, the introduction of some semantic meaning in the aggregation is thus possible for the purpose of controlling the

aggregation phase.

In this paper, we bridge a gap between consensus group decision making and SNA by defining new terminologies and proposing algorithms related to experts preferences, similarity measure, network structure, cohesion subgroups, centrality and consensus measure. We start with forming an undirected weighted preference network structure based on the similarity of experts' preferences. By means of the structural equivalence concept, the closeness of experts' preferences is computed using a similarity function. The utilization of structural equivalence in our preference network produces strong connection to and from most of other experts who have similar preferences. By considering large number of experts and alternatives, and to represent our network pattern based on structural equivalence, the preference network is partitioned into clusters according to their similarity of preferences. It is expected that group consensus can be obtained after consideration of internal and external cohesions, combining both degrees to measure preference homogeneity inside clusters relatively to different clusters. In the event of group consensus being below a minimum acceptable threshold consensus value, a cluster-based feedback mechanism and generation advice is proposed. It is logical to focus on experts within cluster(s) with low consensus contribution and, as mentioned before, to use SNA measures related to influence to design an appropriate feedback mechanism to identify experts and to provide them with recommendations on how to change their preferences to increase consensus. Centrality, one of the important concepts in SNA, is specifically applied in this study in: (i) deciding which experts contribute low to consensus; (ii) determining the leader in the created preference network structure; and (iii) defining a new IOWA operator, the cent-IOWA operator, with ordering of the preferences to aggregate induced via the associated experts' centrality values, in the resolution process to allow for the implementation of the '*soft majority*' concept via a corresponding linguistic quantifier.

The rest of this paper is organized as follows: Section 2 introduces some terminologies on reciprocal preference relations and presents our undirected weighted preference similarity network. Section 3 discusses on the preference similarity network clustering based consensus algorithm: the agglomerative hierarchical clustering with complete linkage is described; cluster consensus with internal and external measures are defined; leading to the introduction of a novel agglomerative clustering based preference network consensus for a group of experts. Section 4 is devoted to the design of an appropriate SNA based feedback mechanism and generation advice procedure. In order to prove the validity of our proposed cluster-based network feedback mechanism, it is proved that when advices are implemented the group consensus increases. This main result, in conjunction with the bounded property of group consensus, guarantee the convergence to sufficient group agreement by the proposed preference similarity network clustering based consensus algorithm. Section 5 presents a resolution process with the introduction of the new cent-IOWA operator in the aggregation phase. In Section 6 an integral view of the general representation of the proposed preference similarity network structural equivalence clustering based consensus decision making model is presented, comparative evaluations with an existing literature study are carried out and an analysis of the main advantages of our proposed model is drawn. Finally, conclusions are pointed out in Section 7.

2. Preference Similarity Network

This section describes concepts and terminology regarding reciprocal preference relations as needed throughout the rest of the paper to derive a new similarity measure based on expert's preferences to represent structural equivalence in their relations.

2.1. Reciprocal Preference Relation

Consider a group of experts, $E = \{e^1, e^2, \dots, e^m\}$, give their opinions towards a finite set of alternatives, $X = \{x_1, x_2, \dots, x_n\}$ ($n > 2$). We assume that each expert expresses his/her

opinion on X by means of pairwise comparisons.

Definition 1. Let X be a non empty set. A fuzzy binary relation R on X is a fuzzy subset of the Cartesian product $X \times X$ characterized by a membership function $\mu_R : X \times X \rightarrow [0, 1]$, where $\mu_R(x_i, x_j) = r_{ij}$ represents the strength of the relation between x_i and x_j .

Definition 2. A reciprocal preference relation on X is a fuzzy binary relation P where the preference intensity of alternative x_i over alternative x_j , $\mu_P(x_i, x_j) = p_{ij}$, verifies $\mu_P(x_i, x_i) = 0.5 \forall x_i \in X$ and $p_{ij} + p_{ji} = 1, \forall x_i, x_j \in X$.

According to Definition 2, an expert not only declares his/her preference on alternative x_i over x_j , but also establishes the intensity of preference by providing the value of p_{ij} . The higher p_{ij} , the higher the preference intensity of alternative x_i over x_j . The associated semantic for the unit interval of a reciprocal preference relation is assumed to be as follows:

$$p_{ij} = \begin{cases} 0 & \text{if } x_j \text{ is definitely preferred to } x_i \\ p_{ij} \in [0, 0.5] & \text{if } x_j \text{ is preferred to } x_i \\ 0.5 & \text{if } x_i \text{ and } x_j \text{ are equally preferred (indifference)} \\ p_{ij} \in [0.5, 1] & \text{if } x_i \text{ is preferred to } x_j \\ 1 & \text{if } x_i \text{ is definitely preferred to } x_j \end{cases}$$

Let $\mathbb{P}_{n \times n}$ denote the set of $n \times n$ matrices P constructed from all reciprocal preference relations on X :

$$P = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{22} & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \dots & p_{nn} \end{bmatrix}$$

verifying: $0 \leq p_{ij} \leq 1$ and $p_{ij} + p_{ji} = 1$ for $i, j \in \{1, 2, \dots, n\}$.

Notice that a reciprocal preference relation can also be mathematically represented by means of a vector known as the *intensity preference vector* [49].

Definition 3. The intensity preference vector of a reciprocal preference relation $P = (p_{ij})_{n \times n} \in \mathbb{P}_{n \times n}$ is the vector of dimension $n(n-1)/2$, $V \in \mathbb{R}^{n(n-1)/2}$, with components the elements above its main diagonal:

$$\begin{aligned} V &= (p_{12}, p_{13}, \dots, p_{1n}, p_{23}, \dots, p_{2n}, \dots, p_{(n-1)n}) \\ &= (v_1, v_2, \dots, v_r, \dots, v_{n(n-1)/2}). \end{aligned}$$

The reciprocity property allows the use of the preference values below the main diagonal of P as components of its intensity preference vector:

$$V_{\text{lower}} = (p_{21}, p_{31}, \dots, p_{n1}, p_{32}, \dots, p_{n2}, \dots, p_{n(n-1)}).$$

Remark: Representation of reciprocal preference relations in terms of preference intensities in fuzzy set theory is referred as reciprocal fuzzy preference relations, which are a particular case of (weakly) complete fuzzy preference relations, i.e. fuzzy preference relations satisfying $p_{ij} + p_{ji} \geq 1, \forall i, j$. However, reciprocal preference relations in probabilistic choice theory are considered as probabilistic binary preference relations [3].

Example 1. In order to demonstrate our proposed model and validate our results, we use the existing numerical example from Chu et al. [27]. Eight (8) experts, $E = \{e_1, e_2, \dots, e_8\}$, provide their judgements on a set of six (6) alternatives, $X = \{x_1, x_2, \dots, x_6\}$, in terms of the following reciprocal preference relations:

$$\begin{aligned}
P^1 &= \begin{bmatrix} 1 & 0.4 & 0.2 & 0.6 & 0.7 & 0.8 \\ 0.6 & 1 & 0.1 & 0.6 & 0.9 & 0.7 \\ 0.8 & 0.9 & 1 & 0.3 & 0.1 & 0.1 \\ 0.4 & 0.4 & 0.7 & 1 & 0.5 & 0.2 \\ 0.3 & 0.1 & 0.9 & 0.5 & 1 & 0.7 \\ 0.2 & 0.3 & 0.9 & 0.8 & 0.3 & 1 \end{bmatrix} & P^2 &= \begin{bmatrix} 1 & 0.3 & 0.3 & 0.5 & 0.6 & 0.6 \\ 0.7 & 1 & 0.4 & 0.7 & 0.2 & 0.3 \\ 0.7 & 0.6 & 1 & 0.5 & 0.4 & 0.2 \\ 0.5 & 0.3 & 0.5 & 1 & 0.6 & 0.7 \\ 0.4 & 0.8 & 0.6 & 0.4 & 1 & 0.4 \\ 0.4 & 0.7 & 0.8 & 0.3 & 0.6 & 1 \end{bmatrix} \\
P^3 &= \begin{bmatrix} 1 & 0.6 & 0.6 & 0.6 & 0.1 & 0.4 \\ 0.4 & 1 & 0.3 & 0.6 & 0.3 & 0.6 \\ 0.4 & 0.7 & 1 & 0.6 & 0.1 & 0.6 \\ 0.4 & 0.4 & 0.4 & 1 & 0.7 & 0.6 \\ 0.9 & 0.7 & 0.9 & 0.3 & 1 & 0.2 \\ 0.6 & 0.4 & 0.4 & 0.4 & 0.8 & 1 \end{bmatrix} & P^4 &= \begin{bmatrix} 1 & 0.3 & 0.2 & 0.5 & 0.7 & 0.7 \\ 0.7 & 1 & 0.3 & 0.7 & 0.3 & 0.5 \\ 0.8 & 0.7 & 1 & 0.7 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.3 & 1 & 0.8 & 0.7 \\ 0.3 & 0.7 & 0.7 & 0.2 & 1 & 0.6 \\ 0.3 & 0.5 & 0.7 & 0.3 & 0.4 & 1 \end{bmatrix} \\
P^5 &= \begin{bmatrix} 1 & 0.6 & 0.3 & 0.6 & 0.6 & 0.7 \\ 0.4 & 1 & 0.1 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.7 & 1 & 0.3 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.7 & 1 & 0.5 & 0.2 \\ 0.4 & 0.2 & 0.7 & 0.5 & 1 & 0.7 \\ 0.3 & 0.6 & 0.8 & 0.8 & 0.3 & 1 \end{bmatrix} & P^6 &= \begin{bmatrix} 1 & 0.3 & 0.1 & 0.5 & 0.7 & 0.6 \\ 0.7 & 1 & 0.4 & 0.7 & 0.2 & 0.4 \\ 0.9 & 0.6 & 1 & 0.5 & 0.4 & 0.2 \\ 0.5 & 0.3 & 0.5 & 1 & 0.6 & 0.7 \\ 0.3 & 0.8 & 0.6 & 0.4 & 1 & 0.4 \\ 0.4 & 0.6 & 0.8 & 0.3 & 0.6 & 1 \end{bmatrix} \\
P^7 &= \begin{bmatrix} 1 & 0.7 & 0.4 & 0.6 & 0.2 & 0.6 \\ 0.3 & 1 & 0.3 & 0.7 & 0.3 & 0.8 \\ 0.6 & 0.7 & 1 & 0.6 & 0.1 & 0.6 \\ 0.4 & 0.7 & 0.4 & 1 & 0.7 & 0.6 \\ 0.8 & 0.7 & 0.9 & 0.3 & 1 & 0.2 \\ 0.2 & 0.4 & 0.4 & 0.4 & 0.8 & 1 \end{bmatrix} & P^8 &= \begin{bmatrix} 1 & 0.4 & 0.4 & 0.3 & 0.5 & 0.6 \\ 0.6 & 1 & 0.3 & 0.7 & 0.3 & 0.4 \\ 0.6 & 0.7 & 1 & 0.7 & 0.2 & 0.3 \\ 0.7 & 0.3 & 0.3 & 1 & 0.8 & 0.7 \\ 0.5 & 0.7 & 0.8 & 0.2 & 1 & 0.6 \\ 0.4 & 0.6 & 0.7 & 0.3 & 0.4 & 1 \end{bmatrix}
\end{aligned}$$

The corresponding intensity preference vectors, $\{V^1, \dots, V^8\}$, are presented below:

$$\begin{aligned}
V^1 &= (0.4, 0.2, 0.6, 0.7, 0.8, 0.1, 0.6, 0.9, 0.7, 0.3, 0.1, 0.1, 0.5, 0.2, 0.7); \\
V^2 &= (0.3, 0.3, 0.5, 0.6, 0.6, 0.4, 0.7, 0.2, 0.3, 0.5, 0.4, 0.2, 0.6, 0.7, 0.4); \\
V^3 &= (0.6, 0.6, 0.6, 0.1, 0.4, 0.3, 0.6, 0.3, 0.6, 0.6, 0.1, 0.6, 0.7, 0.6, 0.2); \\
V^4 &= (0.3, 0.2, 0.5, 0.7, 0.7, 0.3, 0.7, 0.3, 0.5, 0.7, 0.3, 0.3, 0.8, 0.7, 0.6); \\
V^5 &= (0.6, 0.3, 0.6, 0.6, 0.7, 0.1, 0.7, 0.8, 0.4, 0.3, 0.3, 0.2, 0.5, 0.2, 0.7); \\
V^6 &= (0.3, 0.1, 0.5, 0.7, 0.6, 0.4, 0.7, 0.2, 0.4, 0.5, 0.4, 0.2, 0.6, 0.7, 0.4); \\
V^7 &= (0.7, 0.4, 0.6, 0.2, 0.6, 0.3, 0.7, 0.3, 0.8, 0.6, 0.1, 0.6, 0.7, 0.6, 0.2); \\
V^8 &= (0.4, 0.4, 0.3, 0.5, 0.6, 0.3, 0.7, 0.3, 0.4, 0.7, 0.2, 0.3, 0.8, 0.7, 0.6).
\end{aligned}$$

2.2. Undirected Weighted Preference Similarity Network with Structural Equivalence

By using the information from the experts' evaluations toward the alternatives, i.e. the preference intensity vectors, an *expert network structure* is possible to be constructed based on a measurement of preference similarities. The preference similarity between expert e^p and e^q is the same as the preference similarity between expert e^q and e^p , so the weight attached to the tie connecting expert e^p to expert e^q will be identical to the weight attached to the tie connecting expert e^q to expert e^p . Thus, the preference similarity network will be a *nondirectional relation* with the set of *nodes representing the set of experts*, $E = \{e^1, e^2, \dots, e^m\}$, the set of *ties between nodes indicating the similarity relation* between pair of experts, $T = (t_{12}, t_{13}, \dots, t_{1n}, t_{23}, \dots, t_{2m}, \dots, t_{(m-1)m})$, and the *set of weights* attached to the set of ties *representing the strength of the similarity relation*, $S = (s_1, s_2, \dots, s_{m(m-1)/2})$. Formally, the undirected weighted preference similarity network is formulated as below:

Definition 4. An undirected weighted preference similarity network is an ordered triple, $G_S = \langle E, T, S \rangle$ comprising a set of nodes E connected by a set of ties T with a set of preference similarity weights S attached to T .

In order to derive a preference similarity network from a set of experts' reciprocal preference relations on a set of alternatives X , set S in G_S is to be defined. In SNA, similarity between nodes of a network is based on the concept of *structural equivalence*. By definition [50], two experts are structurally equivalent if both of them are similarly tied to and from most of the other experts in the network; in other words, they share the same neighbours. In our context, the structural equivalence concept will represent the idea of experts having similar preferences with other experts. Thus, the structural equivalence concept will rely on the application of a similarity function on the set of intensity preference vectors representing the opinions of the set of experts. Based on Definition 1, a preference similarity measure can formally be modeled as follows:

Definition 5. Let $\mathbb{V} = \{V^1, V^2, \dots, V^m\}$ be a profile of intensity preference vectors expressed by a set of experts, E , towards a set of alternatives, X . A preference similarity measure on E is a fuzzy subset of $\mathbb{V}(X) \times \mathbb{V}(X)$ with membership function $S : \mathbb{V}(X) \times \mathbb{V}(X) \rightarrow [0, 1]$ verifying the following three properties:

1. $0 \leq S(V^p, V^q) \leq 1$
2. $S(V^p, V^p) = 1$ (reflexive)
3. $S(V^p, V^q) = S(V^q, V^p)$ (symmetric)

Hereinafter, the following notation will be used: $S^{pq} = S(V^p, V^q)$. The higher S^{pq} the more similar the preferences of experts e^p and e^q are and, therefore, the more strongly connected these experts are in the preference similarity network G_S . In practice, a particular similarity function is to be chosen. The most common similarity functions between vectors used in representing *structural equivalence* are those based on three groups of distances [51]: (1) Distances based linear correlation as measured by the Pearson correlation coefficient, where the 'focus is on the pattern (strength and direction of association), rather than the mean and variance as aspects of similarity between actors' [52]; (2) Distances based on the Euclidean distance, which are not sensitive to linear association; (3) Distances based on exact matches such as the Jaccard, Hamming distances that focus on exact matches of vectors. The first two groups of distances are normally used for both binary and valued data, while the third one is only applicable to binary data and therefore not useful in the preference representation framework used in this paper. Notice that the cosine similarity corresponds to the special type of Pearson correlation coefficient when the mean of both vectors are considered as zero, i.e. it is a type of Pearson correlation coefficient not sensitive to the mean value. The cosine similarity function has also been shown to be quite stable in measuring consensus regardless of the number of experts involved, which was not the case for the Euclidean distance [13]. Furthermore, given two vectors it is easy to prove that when their cosine similarity increases (decreases), their Euclidean similarity also increases (decreases). In this paper, for illustration purposes, we will make use of the cosine similarity, although similarity functions based on the Euclidean distance or correlation based similarity functions can also be used, and similar results to the ones presented in this paper will be obtained.

Definition 6. The cosine preference similarity measure between the preferences of experts e^p

and e^q is:

$$CS^{pq} = CS(V^p, V^q) = \frac{\sum_{i=1}^{n(n-1)/2} (v_i^p \cdot v_i^q)}{\sqrt{\sum_{i=1}^{n(n-1)/2} (v_i^p)^2} \cdot \sqrt{\sum_{i=1}^{n(n-1)/2} (v_i^q)^2}}. \quad (1)$$

Example 2 (Continuation of Example 1). Using the cosine preference similarity measure as per Definition 6, the following symmetric preference similarity matrix, $CS = (CS^{pq})$, is constructed:

$$CS = \begin{bmatrix} 1 & 0.833 & 0.762 & 0.877 & 0.973 & 0.844 & 0.810 & 0.846 \\ 0.833 & 1 & 0.873 & 0.979 & 0.871 & 0.991 & 0.886 & 0.966 \\ 0.762 & 0.873 & 1 & 0.875 & 0.805 & 0.844 & 0.984 & 0.908 \\ 0.877 & 0.979 & 0.875 & 1 & 0.890 & 0.983 & 0.900 & 0.983 \\ 0.973 & 0.871 & 0.805 & 0.890 & 1 & 0.866 & 0.835 & 0.876 \\ 0.844 & 0.991 & 0.844 & 0.983 & 0.866 & 1 & 0.876 & 0.952 \\ 0.810 & 0.886 & 0.984 & 0.900 & 0.835 & 0.876 & 1 & 0.914 \\ 0.846 & 0.966 & 0.908 & 0.983 & 0.876 & 0.952 & 0.914 & 1 \end{bmatrix}$$

Figure 1 shows the complete graph network representation of the corresponding undirected weighted preference cosine-similarity network as per Definition 4. Notice that only a few link weights are shown for simplicity purpose.

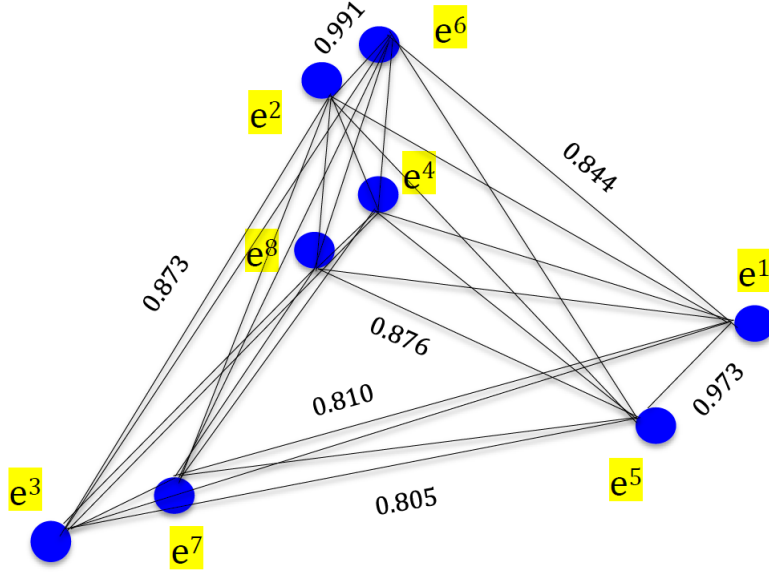


Figure 1: The undirected weighted preference cosine-similarity network for Example 1

Employing a SNA approach, a broad study on network attributes such as pattern of relationships and nodes structure can be explored. The identification of the *most important* actor of a network is one of the primary tasks that can be carried out. In SNA, the centrality concept has been usually used/proposed to identify the most important actor in a network. From a decision making perspective, the expert contributing most to the collective consensus can be considered as a key criterion to be used to identify the most important experts in designing feedback rules, which will be valued by the individual experts in the network, to increase the group

consensus when this is below an acceptable threshold value. Both SNA and decision making with preferences can be linked up by using the centrality concept of the proposed undirected weighted preference similarity network, G_S , to identify the most central expert with respect to agreement/similarity of preferences and therefore the expert with most favoured position in terms of consensus measurement to control the network structure. This is captured in the following definition and will be discussed and exploited further in Section 4.2:

Definition 7. Let $\mathbb{V} = \{V^1, V^2, \dots, V^m\}$ be a profile of intensity preference vectors expressed by a set of experts, E , towards a set of alternatives, X . The expert preference similarity centrality index, $A(e^p)$, is defined as the centrality index in the undirected weighted preference similarity network $G_S = \langle E, T, S \rangle$:

$$A(e^p) = \frac{1}{m-1} \sum_{\substack{q=1 \\ q \neq p}}^m S^{pq} \quad (2)$$

where $p \in \{1, \dots, m\}$.

Example 3 (Continuation of Examples 1–2). As s per Definition 7, the following experts' centrality indices are obtained from the symmetric preference similarity matrix of Example 2:

$$\begin{aligned} A(e^1) &= 0.8492, \quad A(e^2) = 0.9142, \quad A(e^3) = 0.8644, \quad A(e^4) = 0.9266, \\ A(e^5) &= 0.8736, \quad A(e^6) = 0.9082, \quad A(e^7) = 0.8865, \quad A(e^8) = 0.9208. \end{aligned}$$

Clearly, expert e^4 has the highest centrality index, which makes that particular expert to be the most central or most important expert in the constructed undirected weighted preference similarity network.

3. Preference Similarity Network Clustering based Consensus Measure

One approach to representing structural equivalence in a network is using clustering. In general, a *cluster* can be defined as a set of objects which are similar, with objects from different clusters being dissimilar [30]. Different kinds of clustering algorithms have been developed and are therefore available to use, such as partitioning (k-means, k-medoids), hierarchical (agglomerative and divisive), density-based, grid-based algorithms and many more [29]. From a SNA perspective, clustering considers intra-cluster density versus inter-cluster sparsity [53]. According to Wasserman and Faust [54], *hierarchical clustering* is an ideal approach to be utilized in categorizing structurally equivalent experts in a network because it is able to partition them discretely into groups, while providing an explicit procedure and a clear interpretation. In the clustering procedure, pairs of objects (experts) have to be linked in close proximity using *linkage functions*, such as complete link, average link, single link and others. A complete link provides more homogeneous and stable clusters when compared to other linkage functions and it is less susceptible to noise and outliers [55].

3.1. Agglomerative Hierarchical Clustering with Complete Linkage Function

Let S be the similarity function used in the undirected weighted preference similarity network G_S as described in Section 2.2. Since we are using the cosine similarity function in this paper, we use the corresponding cosine-distance, CD , as the distance matrix for the clustering procedure. The *agglomerative hierarchical clustering with complete linkage function* algorithm is described in Algorithm 1 below.

In order to display a hierarchical sequence of clustering solution, a convenient graphic known as *dendrogram* is developed as shown in Figure 2 (Example 4) below. Notice that cutting the

Algorithm 1: Agglomerative hierarchical clustering with complete linkage

Data: A profile of intensity preference vectors, $\mathbb{V} = \{V^1, V^2, \dots, V^m\}$, expressed by a set of experts, $E = \{e^1, \dots, e^m\}$, towards a set of alternatives, $X = \{x_1, \dots, x_n\}$.

Result: A hierarchical sequence of clustering solution: P^m, P^{m-1}, \dots, P^1 .

begin

```
1 | Start the clustering with partition  $P^m = \{C_1, \dots, C_m\}$  where each cluster  $C_p$  has  
  | exactly one element  $e^p$ :  $P^m = \{\{e^1\}, \{e^2\}, \dots, \{e^m\}\} = \{\{C_1\}, \{C_2\}, \dots, \{C_m\}\}$  ;  
  |  $i \leftarrow m$ ;  
  | while  $i > 1$  do  
2 |   Identify clusters  $C_p$  and  $C_q$  in  $P^i = \{C_1, \dots, C_i\}$  with maximal distance ( $D^{pq}$ )  
  |   (complete link);  
3 |   Merge clusters  $C_p$  and  $C_q$  to cluster  $C_k$  ;  
4 |   Build new partition  $P^{i-1}$  by removing  $C_p$  and  $C_q$  and adding cluster  $C_k$  ;  
  |    $i \leftarrow i - 1$ ;  
  | end while  
end
```

dendrogram horizontally at a particular α -level will produce a partition of the experts into clusters equivalent to choosing the number of clusters at that level of the preference similarity matrix, CS , as shown in the 2-dimensional scaling visualisation in Figure 3 (Example 4) below.

Example 4 (Continuation of Examples 1–3). *After the agglomerative hierarchical clustering with complete linkage procedure as given in Algorithm 1 is carried out, the visualisation of clustering outcomes are demonstrated by a dendrogram (Figure 2) and a 2-dimensional scaling of the complete network similarity (Figure 3).*

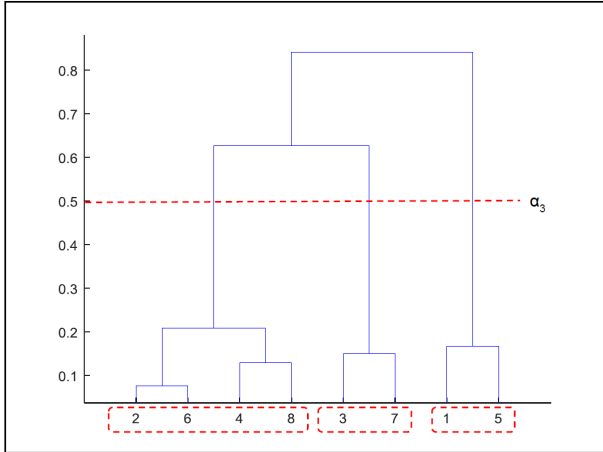


Figure 2: Dendrogram generated from the complete preference similarity network of Example 2

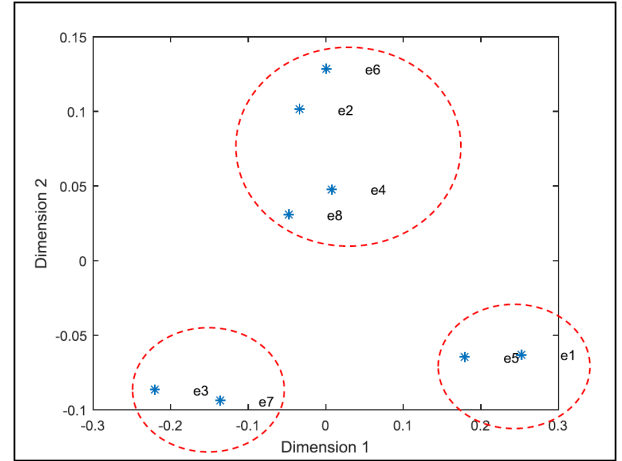


Figure 3: 2-dimensional scaling visualisation of the complete preference similarity network of Example 2

As shown in Figure 2, if the dendrogram is cut at height 0.5, then this corresponds to level 3 (α_3) with experts partitioned into three clusters. In line with the representation in Figure 3, experts are grouped into the three respective clusters: (e^2, e^4, e^6, e^8) , (e^3, e^7) and (e^1, e^5) . In addition, similarity/dissimilarity of pairs of experts can also be observed in Figure 3. As such pairs of experts (e^2, e^6) and (e^4, e^8) are close together because their associated similarity degrees in matrix CS are among the highest, which reflect that they have very similar opinions toward the alternatives. Contrary, experts (e^1, e^3) are very far from each other as their associated

similarity degree are the lowest, showing that they are very dissimilar with respect to their preferences on the alternatives.

3.2. Cluster Consensus with Internal and External Cohesion Measures

As mentioned above, the agglomerative hierarchical clustering provides flexibility in choosing the number of clusters, which can be controlled with the height at which the dendrogram is cut (α -level, Figure 2). This means, though, that there is no predetermined number of clusters at the beginning of the proposed clustering based consensus process. Thus, in our context, it is necessary to devise a criterion of selection of an appropriate α -level clustering. Taking into account that we are interested in achieving a high and acceptable level of consensus among the set of experts, it seems reasonable to choose the agglomerative hierarchical clustering α -level that maximize consensus. Thus, the α -level in the dendrogram with highest degree of group consensus is to be identified. There still remains, however, the issue of how to measure consensus with (agglomerative) hierarchical clustering.

Experts are clustered based on their structural equivalence relation, which means that they are reaching cohesiveness and expect to be connected within their clusters more strongly than with outsider experts. Therefore, it is possible and necessary to measure experts' cluster homogeneity based on their internal and external cohesions, respectively, and combine both to arrive at a collective group cluster-consensus measure.

Let $L = \{\alpha_l ; l = 2, \dots, m - 1\}$ be the set of all distinct α -levels of the agglomerative hierarchical clustering¹. Let $C_l = \{C_{lk} ; k = 1, \dots, l\}$ be the set of clusters at level α_l . Let $\#C_{lk}$ denotes the cardinality of C_{lk} . The cluster internal cohesion index, (δ_{int}) , the cluster external cohesion index, (δ_{ext}) , and cluster-consensus index (δ_{CC}) , are defined as follows:

Definition 8. The α_l -level cluster internal cohesion index of cluster C_{lk} , $\delta_{int}(C_{lk})$, is computed as:

$$\delta_{int}(C_{lk}) = \frac{\sum_{i \in C_{lk}} \sum_{j \in C_{lk}} S^{ij}}{(\#C_{lk})^2}. \quad (3)$$

Definition 9. The α_l -level cluster external cohesion index of cluster C_{lk} , $\delta_{ext}(C_{lk})$, is computed as:

$$\delta_{ext}(C_{lk}) = \frac{\sum_{i \in C_{lk}} \sum_{j \notin C_{lk}} S^{ij}}{\#C_{lk}(n - \#C_{lk})}. \quad (4)$$

Definition 10. The α_l -level cluster consensus index of cluster C_{lk} , $\delta_{CC}(C_{lk})$, is computed as:

$$\delta_{CC}(C_{lk}) = \frac{\#C_{lk} \cdot \delta_{int}(C_{lk})}{n} + \frac{(n - \#C_{lk}) \cdot \delta_{ext}(C_{lk})}{n}. \quad (5)$$

The α_l -level cluster consensus index of cluster C_{lk} can be re-written as:

$$\delta_{CC}(C_{lk}) = \frac{\#C_{lk}(\delta_{int}(C_{lk}) - \delta_{ext}(C_{lk}))}{n} + \delta_{ext}(C_{lk}). \quad (6)$$

The following cases can be distinguished:

¹Levels α_1 and α_m represent the extreme cases of having 1 cluster containing all experts and the initial partition of the agglomerative hierarchical clustering where each member belongs to its own cluster, respectively. In both cases, no clustering technique effectively applies, and consequently are not part of the discussion that follows.

Case 1: $\delta_{int}(C_{lk}) > \delta_{ext}(C_{lk})$. Experts preference similarities are higher internally, which means that they are closer within their group members than the outsider experts, and they are high in homogeneity.

Case 2: $\delta_{int}(C_{lk}) < \delta_{ext}(C_{lk})$. Experts preference similarities are higher externally, meaning that they are closer with outside members compared to their own group members, and they are low in homogeneity.

The underlying concept of clustering the group of experts is based on their preference similarities. Thus, it is expected that Case 1 will prevail in the proposed consensus framework, and it will be used in Section 4 as a criterion to design a feedback mechanism when consensus is not high enough. For the purpose of measuring the group consensus index at each clustering α -level, all clusters' associated consensus indices are combined to arrive at the collective cluster consensus index at that cluster level. This is formally captured in the following definition:

Definition 11. *The α_l -level cluster consensus index of the group of experts E , $\delta_{LC}(l)$, is computed as:*

$$\delta_{LC}(l) = \frac{\sum_{k=1}^l \delta_{CC}(C_{lk})}{l}. \quad (7)$$

The value of $\delta_{LC}(l)$ represents experts' preference homogeneity degree at each clustering α -level, which being based on similarity of preferences is actually measuring the agreement or consensus index between the experts at that cluster level. As mentioned before, our aim is to maximize consensus and therefore the maximum of all the α_l -level cluster consensus indices of the group of experts E is used as the criterion to select the agglomerative hierarchical clustering α_l -level to use in the proposed cluster based consensus model, which is formally captured in the below definition:

Definition 12. *The optimal agglomerative hierarchical clustering level, $\alpha_{\hat{l}}$ -level, is the solution to the following optimization problem*

$$\max_{\alpha_l \in L} \delta_{LC}(l). \quad (8)$$

Notice that the above optimization problem is solvable, i.e. there is a solution in all cases because cardinality of L is finite. However, the solution might not be unique, i.e. there might be more than one α_l -level with same maximum cluster consensus index. In these cases, an additional criterion is required to discriminate further between these α_l -levels. This case resemble the statistical scenario where two sample distributions with different size have the same average value. In these cases, a further statistical measure available to compare these distributions is the coefficient of variation (CV), which takes into account the standard deviation or dispersion of data with respect to the mean value. For equal average values, lower CV is desired as homogeneity of data will be higher. Thus, in our context, the optimal agglomerative hierarchical clustering $\alpha_{\hat{l}}$ -level amongst all the α_l -levels with maximum cluster consensus index will be the one with lowest cluster consensus coefficient of variation, $CCV_{LC}(l)$, as per the following definition:

Definition 13. *The α_l -level cluster consensus coefficient of variation, $CCV_{LC}(l)$ is defined as:*

$$CCV_{LC}(l) = \frac{CSD_{LC}(l)}{\delta_{LC}(l)} \quad (9)$$

where

$$CSD_{LC}(l) = \sqrt{\frac{\sum_{k=1}^l [\delta_{CC}(C_{lk}) - \delta_{LC}(l)]^2}{l}}. \quad (10)$$

In the scenario where there are two or more α_l -levels with same maximum cluster level consensus index and same cluster consensus coefficient of variation, we will choose the lowest α_l -level value as it will result in the lowest number of clusters and consequently, in the event of activating the feedback process (Section 4), a lower number of rounds will be required in order to reach the minimum threshold value of consensus. Summarising, the agglomerative hierarchical cluster consensus index of a group of experts E is thus formally defined as follows:

Definition 14. *The cluster consensus index of a group of experts E is $\delta_{LC}(\hat{l})$, with $\alpha_{\hat{l}}$ -level being the optimal clustering level.*

Let γ be the threshold value for sufficient consensus state. The agglomerative clustering based preference similarity network consensus algorithm with consecutive steps is presented Algorithm 2 below.

Algorithm 2: Agglomerative clustering based preference similarity network consensus

Data: *Dendogram:*
A set of experts: $E = \{e^1, e^2, \dots, e^m\}$;
Set of all different α -levels: $L = \{\alpha_l ; l = 2, \dots, m - 1\}$;
Set of clusters at each α_l -level: $C_l = \{C_{lk} ; k = 1, \dots, l\}$;
Consensus threshold value: $= \gamma$

begin

- 1 Identify experts in each cluster of α_l -level ;
- 2 Compute $\delta_{int}(C_{lk})$ and $\delta_{ext}(C_{lk})$ for each cluster in C_l ;
- 3 Obtain $\delta_{CC}(C_{lk})$ by combining $\delta_{int}(C_{lk})$ and $\delta_{ext}(C_{lk})$ for each cluster in C_l ;
- 4 Calculate $\delta_{LC}(l)$ for all α_l -level in L ;
- 5 Identify optimum agglomerative hierarchical clustering level: $\alpha_{\hat{l}}$ -level;
- 6 **if** $\delta_{LC}(\hat{l}) \geq \gamma$ **then**
| end consensus procedure and apply resolution process;
else
| apply feedback mechanism and advice generation phase;
end if

end

Example 5 (Continuation of Examples 1–4). *The cluster internal (δ_{int}) and external (δ_{ext}) cohesion values, the level cluster consensus indices (δ_{CC}) and cluster consensus indices of the group (δ_{LC}) are calculated and provided in Table 1. At this stage, we observe that the maximum α -level cluster consensus index of the group of experts is 0.906, it is unique and achieved at α -level 4, which makes this clustering level to be the proposed cluster based consensus optimal agglomerative hierarchical clustering level.*

Table 1: The cluster internal and external cohesions, cluster consensus and group consensus indices

α	C	E	δ_{int}	δ_{ext}	δ_{CC}	δ_{LC}
2	1	$e^2, e^3, e^4, e^6, e^7, e^8$	0.940	0.843	0.916	0.897
	2	e^1, e^5	0.986	0.843	0.879	
3	1	e^2, e^4, e^6, e^8	0.982	0.874	0.928	0.899
	2	e^3, e^7	0.992	0.857	0.891	
	3	e^1, e^5	0.986	0.843	0.879	
4	1	e^2, e^6	0.996	0.898	0.922	0.906
	2	e^4, e^8	0.991	0.914	0.933	
	3	e^3, e^7	0.992	0.857	0.891	
	4	e^1, e^5	0.986	0.843	0.879	
5	1	e^2, e^6	0.996	0.898	0.922	0.901
	2	e^4, e^8	0.991	0.914	0.933	
	3	e^3, e^7	0.992	0.857	0.891	
	4	e^1	1	0.849	0.868	
	5	e^5	1	0.874	0.889	
6	1	e^2, e^6	0.996	0.898	0.922	0.899
	2	e^4, e^8	0.991	0.914	0.933	
	3	e^3	1	0.864	0.881	
	4	e^7	1	0.887	0.901	
	5	e^1	1	0.849	0.868	
	6	e^5	1	0.874	0.889	
7	1	e^2, e^6	0.996	0.898	0.922	0.904
	2	e^4	1	0.927	0.936	
	3	e^8	1	0.921	0.931	
	4	e^3	1	0.864	0.881	
	5	e^7	1	0.887	0.901	
	6	e^1	1	0.849	0.868	
	7	e^5	1	0.874	0.889	

4. Preference Similarity Network Clustering based Feedback Mechanism

The cluster-based network feedback mechanism consists of three main phases: (1) identification of experts that contribute less to consensus; (2) identification of a leader in the network; and (3) generation of advice to increase consensus. These phases are described below:

4.1. Identification of Low Contribution to Consensus Experts

The feedback mechanism process is purposely conducted to identify experts contributing less to consensus. The cluster consensus index of the group of experts is the average of the cluster consensus indices of all clusters at the optimum clustering $\alpha_{\hat{l}}$ -level. Thus, experts contributing to consensus less than average are identified, i.e. all clusters at the optimum clustering $\alpha_{\hat{l}}$ -level with cluster consensus index below the cluster consensus index of the group are identified:

$$C_{low} = \left\{ C_{\hat{l}k} \mid \delta_{CC}(k) < \delta_{LC}(\hat{l}) \wedge k = 1, \dots, \hat{l} \right\}. \quad (11)$$

Experts belonging to the clusters in C_{low} are listed as possible low contribution to consensus experts:

$$ec_{low} = \{ ec^y \mid ec^y \in C_{\hat{l}k} \wedge C_{\hat{l}k} \in C_{low} \}. \quad (12)$$

However, not all expert(s) in ec_{low} should be advised to change their preferences because some of them may have centrality indices higher than the cluster consensus index of the group, meaning that they provide sufficient contribution to consensus individually despite of being grouped in cluster(s) with low consensus index. Therefore, only low contribution to consensus experts with additional lower centrality index than the cluster consensus index of the group are identified at this stage:

$$e_{low} = \left\{ e^y \mid e^y \in ec_{low} \wedge A(e^y) < \delta_{LC}(\hat{l}) \right\}. \quad (13)$$

We described the procedure of identifying low contributed experts as in the following steps:

IDENTIFICATION OF LOW CONTRIBUTION TO CONSENSUS EXPERTS	
STEP 1:	Identify set of clusters at the optimum clustering $\alpha_{\hat{l}}$ -level with cluster consensus index below cluster consensus index of the group: C_{low}
STEP 2:	List all experts within cluster(s) identified in previous step: ec_{low}
STEP 3:	Identify expert(s) in ec_{low} with centrality indices lower than the cluster consensus index of the group: e_{low}

Example 6 (Continuation of Example 1–5). *Let us assume that the threshold value of consensus is set at: $\gamma = 0.95$. This means that no consensus state has been reached, and therefore it is necessary to apply the feedback process and implement recommendation advices. We have already established in Example 5 that level 4 is the optimal agglomerative hierarchical clustering level. The level 4 clusters $C_1(e^2, e^6)$, $C_2(e^4, e^8)$, $C_3(e^3, e^7)$ and $C_4(e^1, e^5)$ have cluster consensus indices of 0.922, 0.933, 0.891 and 0.879, respectively. By referring to the consecutive steps of identification of low contribution to consensus experts and the centrality indices of experts of Example 3, we have:*

- STEP 1:* $C_{low} = \{C_3, C_4\}$
STEP 2: $ec_{low} = \{e^1, e^3, e^5, e^7\}$.
STEP 3: $e_{low} = \{e^1, e^3, e^5, e^7\}$, and therefore these experts are considered as low contribution to consensus experts and will be advised to change their preferences for the purpose of increasing the group consensus level.

4.2. Identification of Network Leader

In order to reach sufficient consensus state, identified experts in Section 4.1 receive recommendations on how to change their opinions. We utilize the concept of centrality as defined in Definition 7 to distinguish the most important expert in the network, in such a way that he/she can be appointed as a leader in driving recommendations. Furthermore, as we are aiming to increase the group consensus index, the network leader will be taken from the cluster with highest cluster consensus index. This way, we are integrating within the feedback process both the SNA methodology in that experts will follow the most central expert in the network, and the clustering based consensus methodology in that experts will increase their subsequent contribution to consensus. Summarising, the identification of a leader in the network is carried out by the following consecutive steps:

IDENTIFICATION OF NETWORK LEADER	
STEP 1:	Rank experts according to their centrality indices ($A(e^p)$) in descending order: $R(A(e^p))$
STEP 2:	Identify cluster C_{ik}^* with highest cluster consensus index: $\delta_{CC}(C_{ik}^*) = \max_k \delta_{CC}(C_{ik})$
STEP 4:	Identify the expert e^* in C_{ik}^* with highest centrality index: $A(e^*) = \max_{e^p \in C_{ik}^*} A(e^p)$

Example 7 (Continuation of Examples 1–6). *The experts descending ranking in terms of their experts centrality indices given in Example 3 is: $e^4 \succ e^8 \succ e^2 \succ e^6 \succ e^7 \succ e^5 \succ e^3 \succ e^1$. From Table 1, the highest cluster consensus index at optimal agglomerative hierarchical clustering level (Level 4) corresponds to cluster C_2 . Within cluster C_2 , expert e^4 is identified as the network leader.*

4.3. Generation of Advice

In this phase, identified low contribution to consensus experts will be advised to closely follow the identified network leader's preferences. The simplest form to achieve this is to generate new recommended preference from a linear combination of the original preference of the experts and the target preference of the network leader. Thus, the recommendation rule of change is given as follows:

For all $e^y \in e_{low}$, recommend to change his/her preferences closer to the network leader's preferences using the following linear combination:

$$\tilde{V}^y = \beta \cdot V^y + (1 - \beta) \cdot V^* \quad (14)$$

where V^y is the intensity preference vector of e^y , V^* is the intensity preference vector of the network leader, and $\beta \in [0, 1]$ is a parameter to control the degree of advice.

Notice that when the feedback parameter β takes value 1, the original intensity preference vector (V^y) is kept unchanged, while when β takes value 0 the original intensity preference vector is completely replaced by the network leader's intensity preference vector (V^*). Parameter β value when set by the experts themselves will represent their own acceptable compromise between the group consensus (values of β below 0.5) and their own independence (values of β above 0.5).

In order to prove the validity of the proposed cluster-based network feedback mechanism, the following result proves that when implemented it will lead to an increase in group consensus:

Proposition 1. *Let V^y be the original intensity preference vector of expert e^y , V^* be the network leader's intensity preference vector and $\tilde{V}^y = \beta \cdot V^y + (1 - \beta) \cdot V^*$ be the new intensity preference vector of expert e^y after the recommendation rule is implemented, where $\beta \in [0, 1]$. Then, we have:*

$$CS(\tilde{V}^y, V^*) \geq CS(V^y, V^*) \quad (15)$$

with equality holding if and only if $\beta = 0$.

Proof. There exist points $\mathbf{A}, \mathbf{B}, \mathbf{C} \in \mathbb{R}^{n(n-1)/2}$ such that $V^y = \overrightarrow{\mathbf{OA}}$, $V^* = \overrightarrow{\mathbf{OB}}$ and $\tilde{V}^y = \overrightarrow{\mathbf{OC}}$, which can be represented as in Figure 4 below:

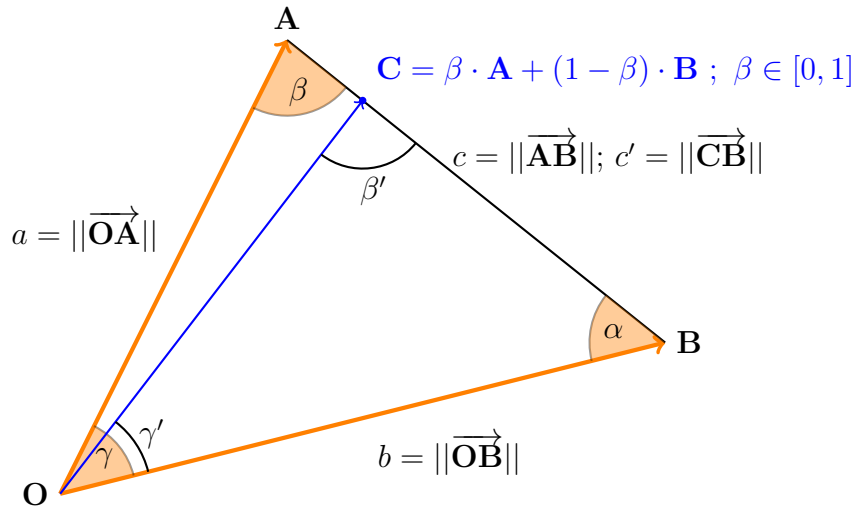


Figure 4: Spatial representation of intensity preference vectors

We have the following:

1. Because coordinates of points \mathbf{A} and \mathbf{B} are greater or equal to zero it is $\gamma \in \left[0, \frac{\pi}{2}\right]$.
2. Clearly, it is $\|\overrightarrow{\mathbf{AB}}\| = c \geq c' = \|\overrightarrow{\mathbf{CB}}\|$, with equality holding if and only if $\beta = 0$.
3. Because $\alpha + \beta + \gamma = \pi$, we have: $\sin \beta = \sin \alpha \cdot \cos \gamma + \cos \alpha \cdot \sin \gamma$.

The law of sines

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

implies that

$$c = b \cdot \frac{\sin \gamma}{\sin \beta}$$

From item 3, we have

$$c = b \cdot \frac{\sin \gamma}{\sin \alpha \cdot \cos \gamma + \cos \alpha \cdot \sin \gamma} = b \cdot \frac{1}{\sin \alpha \cdot \cot \gamma + \cos \alpha}$$

From item 1, we have

$$0 \leq \cot \gamma \leq \cot \gamma' \Leftrightarrow \gamma \geq \gamma'$$

From item 2, we have that $\gamma \geq \gamma'$, with equality holding if and only if $\beta = 0$. Therefore, we have that

$$\cos \gamma \leq \cos \gamma'$$

with equality holding if and only if $\beta = 0$. Finally, by Definition 6 it is concluded that

$$CS(\tilde{V}^y, V^*) \geq CS(V^y, V^*)$$

with equality holding if and only if $\beta = 0$. □

Example 8 (Continuation of Examples 1–7). *For generating advice, we are assuming for illustrative purposes that $\beta = 0.5$. Low contribution to consensus experts (e^1, e^3, e^5, e^7) will be advised to change their original intensity preference vectors to the following ones that take into account the network leader's intensity preference vector ($V^4 = V^*$):*

$$\tilde{V}^1 = (0.35, 0.20, 0.55, 0.70, 0.75, 0.20, 0.65, 0.60, 0.60, 0.50, 0.20, 0.20, 0.65, 0.45, 0.65)$$

$$\tilde{V}^3 = (0.45, 0.40, 0.55, 0.40, 0.55, 0.30, 0.65, 0.30, 0.55, 0.65, 0.20, 0.45, 0.75, 0.65, 0.40)$$

$$\tilde{V}^5 = (0.45, 0.25, 0.55, 0.65, 0.70, 0.20, 0.70, 0.55, 0.45, 0.50, 0.30, 0.25, 0.65, 0.45, 0.65)$$

$$\tilde{V}^7 = (0.50, 0.30, 0.55, 0.45, 0.65, 0.30, 0.70, 0.30, 0.65, 0.65, 0.20, 0.45, 0.75, 0.65, 0.40).$$

Assuming that all low contribution to consensus experts implement the recommended new intensity preference vectors $\tilde{V}^1, \tilde{V}^3, \tilde{V}^5$ and \tilde{V}^7 , new feed-backed reciprocal preference relation matrices $\tilde{P}^1, \tilde{P}^3, \tilde{P}^5$ and \tilde{P}^7 are generated. The second round of the preference similarity network clustering based consensus model is applied, resulting in an increase of the group consensus index for the second round from 0.906 to 0.970, which is greater than the threshold value, γ , meaning that a consensus state has been reached and resolution process can be carried out.

5. Resolution Process with cent-IOWA Operator

When group consensus is sufficient, it means that experts individual preferences have been considered appropriately and the feasible solution obtained in the *resolution process* will give satisfaction to the entire group. The resolution process involves two main procedures: Aggregation and exploitation phases. Experts' opinions will be collectively fused in the aggregation phase and a final ranking of alternatives will be derived in the exploitation phase.

5.1. Aggregation Phase

A useful tool in handling the information fusion is the OWA operator, introduced by Yager [40].

Definition 15. *An OWA operator of dimension n is a function $\phi: \mathbb{R}^n \rightarrow \mathbb{R}$ that has associated a set of weights or weighting vector, $W = (\omega_1, \dots, \omega_n)$, to it verifying $\omega_i \in [0, 1]$ and $\sum_{i=1}^n \omega_i = 1$, with following expression:*

$$\phi(p_1, \dots, p_n) = \sum_{i=1}^n \omega_i \cdot p_{\sigma(i)} \quad (16)$$

being $\sigma: \{1, \dots, n\} \rightarrow \{1, \dots, n\}$ a permutation such that $p_{\sigma(i)} \geq p_{\sigma(i+1)}$, $\forall i = 1, \dots, n-1$, i.e., $p_{\sigma(i)}$ is the i -th highest value in the set $\{p_1, \dots, p_n\}$.

Related to the OWA operator, the concept of *fuzzy majority*, represented by natural language expressions such as ‘most of’, ‘at least half’, ‘as many as possible’, was defined and implemented by means of *quantifier guided linguistic OWA operators* [56] with experts' weights

vector generated via the mathematical formulation of the soft majority concept via an appropriate quantifier membership function Q . For the case of a regular increasing monotone (RIM) quantifier, $Q: [0, 1] \rightarrow [0, 1]$ such that $Q(0) = 0, Q(1) = 1$ and if $x > y$ then $Q(x) \geq Q(y)$, the OWA weights express the proportion of criteria satisfied by an alternative [40]:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), i = 1, \dots, n. \quad (17)$$

Yager [56] also presented the evaluation procedure of the overall satisfaction of ‘ Q important criteria or experts’ (μ_k or e_k) by an alternative x_j . In this case, the weights of the quantifier guided linguistic OWA operator are determined using the following expression:

$$w_i = Q\left(\frac{\sum_{k=1}^i \mu_{\sigma(k)}}{Z}\right) - Q\left(\frac{\sum_{k=1}^{i-1} \mu_{\sigma(k)}}{Z}\right) \quad (18)$$

where $Z = \sum_{k=1}^n \mu_k$ is the total sum of importance and σ is the permutation applied to obtain the ordering of the values to be fused.

The *Induced OWA* (IOWA), which extend the OWA operator, was specifically proposed by Yager and Filev [41] to induce the reordering step of the argument variable upon the magnitude of an additional variable, known as the order inducing variable.

Definition 16. An IOWA operator of dimension n is a function $\Phi_W: (\mathbb{R} \times \mathbb{R})^n \rightarrow \mathbb{R}$, to which a set of weights or weighting vector is associated $W = (\omega_1, \dots, \omega_n)$ such that $\omega_i \in [0, 1]$ and $\sum_i \omega_i = 1$, and that aggregates the set of second arguments of a list of n 2-tuples $\{\langle u_1, p_1 \rangle, \dots, \langle u_n, p_n \rangle\}$ according to the following expression,

$$\Phi_W \{\langle u_1, p_1 \rangle, \dots, \langle u_n, p_n \rangle\} = \sum_{i=1}^n \omega_i \cdot p_{\sigma(i)} \quad (19)$$

being $\sigma: \{1, \dots, n\} \rightarrow \{1, \dots, n\}$ a permutation such that $p_{\sigma(i)} \geq p_{\sigma(i+1)}, \forall i = 1, \dots, n-1$, i.e., $\langle u_{\sigma(i)}, p_{\sigma(i)} \rangle$ is the 2-tuple with $u_{\sigma(i)}$ the i -th highest value in the set $\{u_1, \dots, u_n\}$.

Definition 16 can be used to construct a new aggregation operator based on the centrality index associated with each expert $\{A(e^1), \dots, A(e^m)\}$ as importance values associated to the experts $E = \{e^1, e^2, \dots, e^m\}$ and also as the order inducing variable of an IOWA operator to produce the ordering of the experts’ preference values to be aggregated $\{p_{ij}^1, \dots, p_{ij}^m\}$. This new operator is named as *cent-IOWA operator*:

Definition 17. The *cent-IOWA operator* of dimension m , Φ_w^{cent} , is an IOWA operator whose set of order inducing values is the set of centrality indices of the nodes of a network $\{A(e^1), \dots, A(e^m)\}$.

Using the *cent-IOWA operator*, Φ_w^{cent} , the *collective preference relation* is determined as follows:

$$p_{ij}^c = \Phi_w^{cent}(\langle A(e^1), p_{ij}^1 \rangle, \dots, \langle A(e^m), p_{ij}^m \rangle) \quad (20)$$

being Q the fuzzy linguistic quantifier used to implement the concept of soft majority via the computation of the weighting vector using expression (18). We utilize RIM quantifier $Q(r) = r^{1/2}$ to represent fuzzy linguistic quantifier ‘most of’ as introduced by Yager [56] to implement the concept of “preference of one alternative over another for ‘most of’ the more central experts.”

Example 9. (Continuation of Examples 1–8) The second round of the preference similarity network clustering based consensus model using the set of feed-backed experts' preference relations $(\tilde{P}^1, P^2, \tilde{P}^3, P^4, \tilde{P}^5, P^6, \tilde{P}^7, P^8)$ produces the following new experts' centrality indices:

$$\begin{aligned}\tilde{A}(e^1) &= 0.9539, \quad \tilde{A}(e^2) = 0.9632, \quad \tilde{A}(e^3) = 0.9631, \quad \tilde{A}(e^4) = 0.9762, \\ \tilde{A}(e^5) &= 0.9629, \quad \tilde{A}(e^6) = 0.9605, \quad \tilde{A}(e^7) = 0.9663, \quad \tilde{A}(e^8) = 0.9649.\end{aligned}$$

Notice that the new centrality indices after the feedback process are higher than in the previous round of consensus as given in Example 3, with expert e_4 still having the highest centrality index value.

The aggregation of experts' preference relations $(\tilde{P}^1, P^2, \tilde{P}^3, P^4, \tilde{P}^5, P^6, \tilde{P}^7, P^8)$ using the cent-IOWA operator guided by the fuzzy linguistic quantifier 'most of', with corresponding weighting vector is $\tilde{W} = (0.356, 0.146, 0.112, 0.094, 0.084, 0.075, 0.069, 0.064)$, produces the following collective preference relation:

$$\tilde{P}^c = \begin{bmatrix} 1 & 0.3675 & 0.2600 & 0.4960 & 0.6028 & 0.6558 \\ 0.6325 & 1 & 0.3024 & 0.6926 & 0.3216 & 0.4919 \\ 0.7400 & 0.6976 & 1 & 0.6281 & 0.2757 & 0.3080 \\ 0.5040 & 0.3074 & 0.3719 & 1 & 0.7350 & 0.6538 \\ 0.3972 & 0.6784 & 0.7243 & 0.2650 & 1 & 0.5283 \\ 0.3442 & 0.5081 & 0.6920 & 0.3462 & 0.4717 & 1 \end{bmatrix}$$

5.2. Exploitation Phase

Once aggregation of individual expert preferences into the collective form has been done, the exploitation phase is carried out to determine a *total ranking* of the set of alternatives. The *Quantifier Guided Dominance Degree* (QGDD) [57] based on the use of the OWA operator (Definition 15) guided by the linguistic quantifier Q is applied:

Definition 18. Given a preference relation $P^c = (p_{ij}^c)$ on a finite set of alternatives $X = \{x_1, x_2, \dots, x_n\}$, the quantifier guided dominance degree, $QGDD(x_i)$, quantifies the dominance that an alternative x_i has over all the other alternatives in a fuzzy majority sense as:

$$QGDD(x_i) = \Phi_Q(p_{ij}^c, j = 1, \dots, n, j \neq i) \quad (21)$$

where Φ_Q is an OWA operator guided by the linguistic quantifier Q representing the fuzzy majority concept.

The elements of the set,

$$X^{QGDD} = \{x \mid x \in X, QGDD(x) = \sup_{x \in X} QGDD(x)\} \quad (22)$$

are called the maximum dominance elements of the fuzzy majority of X quantified by Q . In our case, the maximal dominance set quantifies the best alternative chosen is according to 'most of' the central experts in the undirected weighted similarity preference network where they are contributing more to consensus, and the decision made will be agreed by the whole group of expert since the consensus level achieved is already sufficient at this stage of the resolution process.

Example 10 (Continuation of and Finishing Examples 1–9). The dominance guided choice degree of the alternatives using the fuzzy linguistic quantifier 'most of' with the corresponding weighting vector $W = (0.447, 0.185, 0.143, 0.119, 0.106)$ are:

$$QGDD = (0.5469, 0.5673, 0.6155, 0.5984, 0.6002, 0.5484).$$

Therefore, the final ordering of alternatives is:

$$x_3 \succ x_5 \succ x_4 \succ x_2 \succ x_6 \succ x_1.$$

This makes alternative x_3 to be the best alternative solution of consensus according to ‘most of’ the central experts in the network.

6. Preference Similarity Network Structural Equivalence Clustering based Consensus Group Decision Making Model Framework

This section focuses on the general representation of the proposed model, comparative analysis from which conclusions are drawn and explained, and the analysis of the proposed model with respect to previous studies in the literature.

6.1. General Representation of the Proposed Model

This section provides the overall model representation with all its components developed in the previous sections together, so that readers are allow to have a general view of the proposed preference similarity network structural equivalence clustering based consensus group decision making model framework, as shown in Figure 5. Specifically, the model consists of four main phases: (1) development of preference similarity network; (2) preference similarity network clustering based consensus measure; (3) preference similarity network clustering based feedback mechanism; and (4) the resolution process with the cent-IOWA operator.

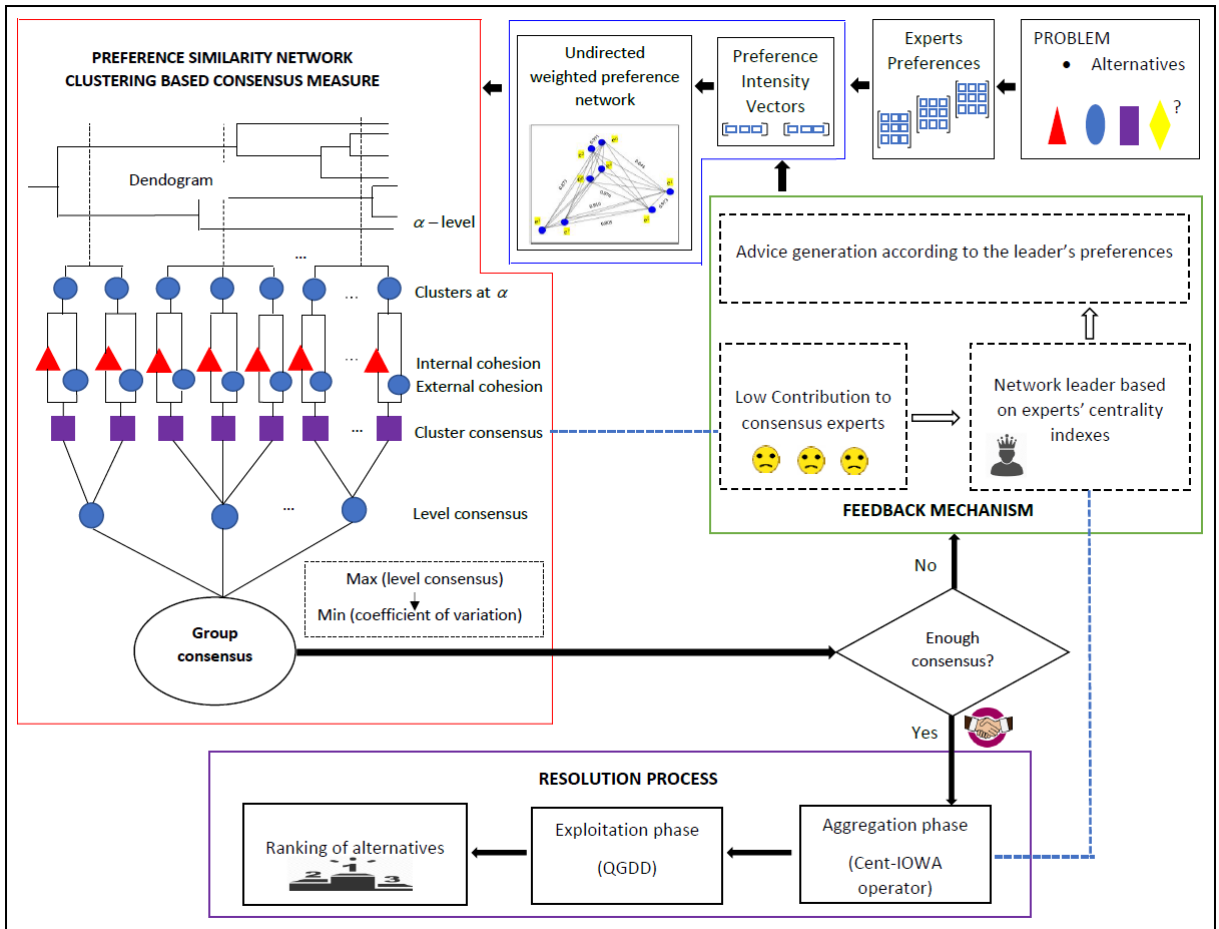


Figure 5: A framework of preference similarity network structural equivalence clustering based consensus group decision making model

The description of each phase is briefly explained in the following:

- *Phase 1 – Development of preference similarity network*: By referring to Section 2, a novel undirected weighted preference network similarity measure is built on the reciprocal preference relations provided by a group of experts on a set of alternatives. Preference intensity vectors are generated and the similarity of preference network is determined based on the application of an appropriate similarity function, with the cosine one chosen in this paper for illustrative purposes. Similar results to the obtained with the cosine preference similarity function are obtained if a different similarity function were used, such as the Euclidean distance based similarity (refer to Proposition 1). The proposed similarity measure expresses the strength of experts' connections sharing most similar preferences, known as structural equivalence relation. The undirected weighted preference similarity network is then formed according to the similarity preference matrix, where the nodes represent the experts, and the weights attached to the links connecting them providing the degree of preference similarity relations.
- *Phase 2 – Preference similarity network clustering based consensus measure*: As presented in Section 3, an undirected weighted preference similarity network structure is utilized in the agglomerative hierarchical clustering with complete linkage function for the purpose of partitioning experts into subgroups based on their preference similarities. A dendrogram is generated to display the clustering solution at all α -levels. Since cluster experts are structurally equivalent, clustering technique based on the complete linkage makes them homogeneous, meaning experts to be strongly connected to each other within the cluster in comparison to experts in different clusters. By combining clusters' internal and external cohesions, the α -level clusters consensus index and the α -level cluster group consensus index are defined and measured. Finally, the cluster consensus index of the group is defined as the highest of all the α -level cluster group consensus indices, in other words it is the cluster consensus index of the group at the optimal clustering level.
- *Phase 3 – Preference similarity network clustering based feedback mechanism*: When the cluster group consensus is not high enough, i.e. it is lower than a threshold value representing consensus reaching state, a feedback mechanism phase (see Section 4) that integrates both SNA and clustering methodologies is designed, with three main consecutive steps: identification of low contribution to consensus experts; network leader identification; and advice generation. Basically, the feedback steps demonstrate a recommendation system that focuses on the experts who contribute less to consensus and that are guided by the most central expert of those contributing most to consensus. It has been proved that when generated advices are implemented, consensus increases. In fact, because consensus is bounded this result guarantees the feedback process convergence to consensus reaching state.
- *Phase 4 – Resolution process with the introduction of the cent-IOWA operator*: In Section 5, all expert preferences are fused into the collective preferences by using a new aggregation operator (cent-IOWA) that induces the ordering of the preferences to aggregate based upon the centrality index associates with each of expert. The exploitation procedure is then carried out by implementing the OWA quantifier guided dominance degree (QGDD) to derive the final ranking of alternatives from which the maximum dominance element is chosen as the solution of consensus for 'most of' the central experts in the network.

6.2. Comparative Evaluations of the Proposed Model

For the validation purpose, we revisit Chu's [27] work and make comparisons on the impact of social network connections in group decision making. We focus on three main areas: (1) Aggregation weighting vector; (2) collective preferences; and (3) ranking of alternatives. From Table 2 results comparison we draw the following discussion:

Table 2: Comparative results on the social network connections in group decision making

			Weights vector	Collective preferences					Ranking of alternatives	
No network connection [27]		(0.124, 0.123, 0.126, 0.125, 0.127, 0.122, 0.125, 0.128)	1	0.4579	0.3145	0.5289	0.5122	0.6248	$x_5 \succ x_6 \succ x_4 \succ x_3 \succ x_1 \succ x_2$	
			0.5421	1	0.2760	0.6800	0.4150	0.5110		
			0.6982	0.6986	1	0.5187	0.2393	0.3074		
			0.4711	0.3000	0.4813	1	0.6523	0.5503		
			0.4878	0.5850	0.7607	0.3477	1	0.4698		
			0.3502	0.5040	0.6926	0.4497	0.5302	1		
Directed network [27]		(0.408, 0.150, 0.103, 0.089, 0.079, 0.061, 0.061, 0.048)	1	0.3702	0.2540	0.4998	0.6169	0.6570	$x_4 \succ x_3 \succ x_5 \succ x_2 \succ x_6 \succ x_1$	
			0.6298	1	0.3006	0.7056	0.3579	0.4706		
			0.7460	0.6988	1	0.5878	0.2725	0.2720		
			0.5002	0.3132	0.4122	1	0.7144	0.6297		
			0.3831	0.6421	0.7275	0.2856	1	0.5223		
			0.3336	0.5388	0.7280	0.3703	0.4777	1		
Undirected preference network (Not enough consensus)	weighted similarity	(0.360, 0.149, 0.113, 0.095, 0.082, 0.073, 0.067, 0.061)	1	0.3958	0.2821	0.4985	0.5704	0.6421	$x_5 \succ x_3 \succ x_4 \succ x_2 \succ x_6 \succ x_1$	
			0.6042	1	0.2940	0.6872	0.3523	0.4892		
			0.7179	0.6914	1	0.5899	0.2639	0.3044		
			0.5015	0.3456	0.4101	1	0.7033	0.6181		
			0.4296	0.6477	0.7361	0.2967	1	0.5122		
			0.3415	0.5272	0.6956	0.3819	0.4878	1		
Undirected preference network (Enough consensus)	weighted similarity	(0.356, 0.146, 0.112, 0.094, 0.084, 0.075, 0.069, 0.064)	1	0.3675	0.2600	0.4960	0.6028	0.6558	$x_3 \succ x_5 \succ x_4 \succ x_2 \succ x_6 \succ x_1$	
			0.6325	1	0.3024	0.6926	0.3216	0.4919		
			0.7400	0.6976	1	0.6281	0.2757	0.3080		
			0.5040	0.3074	0.3719	1	0.7350	0.6538		
			0.3972	0.6784	0.7243	0.2650	1	0.5283		
			0.3442	0.5081	0.6920	0.3462	0.4717	1		

- *Weighting vector*: The no network connection weighting vector is obviously different from the directed network and both of the undirected weighted preference similarity networks. This is because the weights are directly assumed by the authors in [27] since the experts are considered completely independent one from each other. Otherwise, weights for experts linked by networks are generally obtained from experts' centrality measures. For the directed network in [27], I-IOWA operator based on in-degree and out-degree centrality indices was developed with experts weights being higher the higher their associated centrality indices. In the proposed undirected network structure, the cent-IOWA operator does not consider the in-degree or out-degree indices but the experts' preference similarity based on the structural equivalence concept to generate the centrality indices. The formation of the proposed preference similarity network implies the contribution of experts to consensus. Thus, when measuring consensus as well as in the proposed feedback mechanism, the similarity of opinion between experts are increased accordingly because some experts need to change their preferences closer to the network leader in order to achieve sufficient agreement. This might increase the centrality indices, which seems to be the explanation behind the slightly difference between the weighting vectors of the undirected weighted preference similarity network without consensus and with sufficient consensus. We can conclude that social network provides an impact on the generation of experts' associated weights and that the proposed novel preference network structure based on similarities between nodes provides similar weighting vectors in the different rounds of consensus, with slight differences reflecting the increase in experts centrality indices.
- *Collective preferences*: The collective preference relations are all different due to the difference in the aggregation operator used but also due to the difference in the weighting vectors implemented. The slight difference in the weighting vectors of the proposed

undirected weighted preference similarity network without consensus and with sufficient consensus does not explain the difference between the collective preference relations derived in these cases but the changes in the experts (e^1, e^3, e^5, e^7) individual preferences introduced by the advice generation rule in feedback mechanism. Other than the effect of social network in generating weighting vector, the aggregation of preferences from individual expert to collective one are also influenced by the consensus process, with the additional positive effect of increasing group consensus.

- *Ranking of alternatives:* It is clearly shown that the ranking of alternatives when social network does not play a role is totally different to the other three rankings, implying that the social network connections truly impact the decision making process. However, the directed network in [27] and both the proposed undirected weighted preference similarity network without consensus and with sufficient consensus rank the same last three alternatives ($x_2 \succ x_6 \succ x_1$) and slightly differ in the ordering of the first three alternatives. The best alternative solution for the directed and undirected weighted preference similarity network (not enough consensus) are x_4 and x_5 , respectively, with the difference due to the different ranking approaches they use. The derivation of the priority weighting vector by Fedrizzi and Brunelli [58] was applied in the directed network connection in [27], while the dominance guided choice degree with fuzzy linguistic quantifier ‘most of’ was applied in the proposed undirected weighted preference similarity network (not enough consensus). Again, the ranking of alternatives for the proposed undirected weighted preference similarity networks with enough consensus is slightly different to the undirected weighted preference similarity networks (not enough consensus) because the consensus feedback process introduced changes in the individual preferences of half of the experts leading to the acceptance of the decision by the group as a whole. We can say that the proposed cluster based consensus measure gives some flexibility to the experts to revise their opinion for the sake of achieving sufficient group agreement and obtaining good solution to satisfy them all.

In Table 3, we provide another comparative results for non-clustering and clustering consensus group decision making models based on our proposed algorithm. As shown in Example 1, there are eight experts giving evaluation towards a set of six alternatives. We consider non-clustering consensus decision making model by taking maximum α -level (Level 8) as optimum consensus level, where each expert is individually positioned in its own cluster. On the other hand, the clustering based model follows the optimum consensus level (Level 4) as in Example 4. For non-clustering based model, the internal cohesions are maximum ($\delta_{int} = 1$) because they are closer with themselves (self-consensus), while the clustering based model has reliable internal cohesions because every cluster has more than one member and they are unitedly measured. Other than that, the clustering based model shows that experts are closer within their group members than the outsiders because their $\delta_{int} > \delta_{ext}$, proving that they are higher in homogeneity. Similar conclusion cannot be drawn for non-clustering model because each expert does not have group member and only relies on their self-consensus. By comparing experts/clusters consensus degrees, we can see that most of the experts of non-clustering based model have higher in degree values because they are affected by maximum internal cohesions. However, both approaches give the same consensus degrees for the first round of consensus process, the same low contributed consensus experts and leader of the network. A second round of consensus process is carried out for both methods because none of them achieved sufficient consensus degrees (consensus degree lower than the threshold). The consensual state is reached in the second round with close global consensus values but higher in the proposed clustering based model with optimum consensus level.

Summarizing Table 3, we can say that non-clustering and clustering based consensus decision

Table 3: Comparison results of non-clustering and clustering based consensus group decision making models

Elements	Non-clustering based model	Clustering based model
Optimal consensus level	8	4
Internal cohesions	$\delta_{int}(e^1) = 1, \delta_{int}(e^2) = 1,$ $\delta_{int}(e^3) = 1, \delta_{int}(e^4) = 1,$ $\delta_{int}(e^5) = 1, \delta_{int}(e^6) = 1,$ $\delta_{int}(e^7) = 1, \delta_{int}(e^8) = 1$	$\delta_{int}(C_1) = 0.996, \delta_{int}(C_2) = 0.991,$ $\delta_{int}(C_3) = 0.992, \delta_{int}(C_4) = 0.986.$
External cohesions	$\delta_{ext}(e^1) = 0.849, \delta_{ext}(e^2) = 0.914,$ $\delta_{ext}(e^3) = 0.864, \delta_{ext}(e^4) = 0.927,$ $\delta_{ext}(e^5) = 0.874, \delta_{ext}(e^6) = 0.908,$ $\delta_{ext}(e^7) = 0.887, \delta_{ext}(e^8) = 0.921$	$\delta_{ext}(C_1) = 0.898, \delta_{ext}(C_2) = 0.914,$ $\delta_{ext}(C_3) = 0.857, \delta_{ext}(C_4) = 0.843.$
Cluster(s)/expert(s) consensus	$\delta_{CC}(e^1) = 0.868, \delta_{CC}(e^2) = 0.925,$ $\delta_{CC}(e^3) = 0.881, \delta_{CC}(e^4) = 0.936,$ $\delta_{CC}(e^5) = 0.889, \delta_{CC}(e^6) = 0.920,$ $\delta_{CC}(e^7) = 0.901, \delta_{CC}(e^8) = 0.931$	$\delta_{CC}(C_1) = 0.922, \delta_{CC}(C_2) = 0.933,$ $\delta_{CC}(C_3) = 0.891, \delta_{CC}(C_4) = 0.879.$
Consensus degree (1st round)	0.906	0.906
Low contributed consensus expert(s)	e^1, e^3, e^5, e^7	$C_3 = \{e^3, e^7\}$ and $C_4 = \{e^1, e^5\}$
Network leader	e^4	e^4
Consensus degree (2nd round)	0.968	0.970

making models provide consistent results with each other. However, clustering based model is more manageable than the non-clustering one because it presents all results in terms of groups. The strength of the clustering method with respect to the non-clustering is clearly shown when large number of experts and alternatives are involved. Tediousness of analysis can be reduced and inaccuracy can be prevented.

6.3. Analysis of the Proposed Model

The main advantages of the proposed model and its differences with respect to previous studies in the literature are presented as follows:

- (i) The preference network is constructed by incorporating generated weights from similarity of experts' preferences based on the structural equivalence concept [50]. This measure conceptually differs from previous work done in most similarity-based consensus models [19, 21] because they do not consider any network criterion, such as the connected ties and structural classes.
- (ii) Our proposal provides an alternative solution to expert weights derivation, which overcome the assumption that the weights of experts are known beforehand [13]. Meanwhile, the absence of weighting vector does not necessarily mean that all experts are equally important. Indeed, once experts provide opinions, their preference similarities can be used to derive weighting values. This is an advantage in measuring consensus, as more importance will be given to experts with higher centrality indices.
- (iii) The use of the centrality concept to determine the most important node (expert) in the network is defined and implemented differently from [4] and [27]. In the proposed model, the identification procedure of a leader based on the centrality index is introduced as an additional step in the feedback mechanism with a positive impact in increasing the group consensus state when experts are advised to get closer to the leader's preferences.

- (iv) The cluster-based consensus model based on the proposed defined internal and external cohesions, cluster consensus and level consensus is one of the first efforts in deriving cluster-based group consensus model in decision making. Previous works done by Garcia-Lapresta and Perez-Roman [35–37], Abel et al. [38] and Li et al. [39] focused on different contexts of clustering-based consensus.
- (v) A new cent-IOWA aggregation operator is introduced, which produces ordering of the argument values based upon the centrality index associated with each expert and provides an alternative information fusion approach in the resolution process. The proposed operator is inspired by the concept of IOWA [41] operator, and it allows for the implementation of the concept of soft majority in the consensus model.

7. Conclusion

This paper proposed an undirected weighted preference network based on experts' preference similarities and, accordingly, a cluster-based consensus measure with feedback mechanism algorithm and resolution process, in conjunction with a new aggregation operator and soft majority concept. Mainly, we bridge a gap between SNA and consensus-based decision making by: (1) developing an undirected weighted preference network according to experts' preference similarities; (2) representing structurally equivalent experts using agglomerative hierarchical clustering algorithm; (3) applying the centrality concept in determining a network leader to drive advices in the feedback mechanism, and in constructing a new IOWA aggregation operator (cent-IOWA). The implementation of the proposed model guarantees convergence to sufficient group agreement, which facilitates the feasibility of the final alternative solution. Comparative evaluations have been presented in order to analyse the impact of social network connections in group decision making, the differences/similarities of non-clustering and clustering based model in achieving consensus and the main advantages of the proposed model to previous studies in the literature, respectively.

It is well-known that knowledge contributions on decision making and SNA provide huge area of studies to be explored, therefore continuous works especially on consensual reaching process with application of SNA concepts need to be taken into account. Moreover, clustering techniques as used in this paper have the potential to benefit decision making processes with big data arising from a high number of experts, criteria and/or alternatives, which will be further explored in future.

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